ADAPTIVE BEHAVIOR AND COORDINATION FAILURE

John B. Van Huyck, Joseph P. Cook, and Raymond C. Battalio

Revised March 1996

TAMU Economic Research Laboratory
Department of Economics
Texas A&M University
College Station, TX 77843-4228

Abstract: We use the experimental method to study people's adaptive behavior in a generic game with multiple Pareto ranked equilibria. The experiment was designed to discover if behavior diverged at the separatrix predicted by the fictitious play dynamic. The equilibrium selected was sensitive to small differences in initial conditions as predicted. The experiment provides some striking examples of coordination failure growing from small historical accidents.

Key Words: adaptive learning, fictitious play, path dependence, coordination failure, predictive success.

JEL classification: c720, c920.

Acknowledgments: Artie Powell made valuable comments on the experimental design and helped run the experiments reported in this paper. The National Science Foundation provided financial support.

I. INTRODUCTION

The power of the equilibrium method derives from the ability to abstract from the dynamic process that produces mutually consistent behavior and to abstract from the historical accident that initiated the process. This ability depends on an appeal to the long run: a time when adaptive behavior will have converged to a unique stable equilibrium, see Lucas (1987). In this paper, we consider two related problems with this traditional defense of the equilibrium method: non-convergence and non-uniqueness.[1]

First, models of adaptive behavior do not guarantee convergence to Nash equilibria in general. For example, Cournot's (1960) myopic best response dynamic and Brown's (1951) fictitious play dynamic can lead to cycles or chaos. Hence, an open question is whether we should expect strategic behavior to converge to an outcome that satisfies a mutual consistency condition, like Nash equilibrium, or to an outcome that does not, like rationalizability.

Second, multiple equilibria undermine the usefulness of an analysis that abstracts from historical accident and dynamic process. Multiple equilibria arise in many economic contexts. For example, multiple Pareto ranked equilibria arise in both macroeconomic models with production, search, or trading externalities and microeconomic models of monopolistic competition, technology adoption and diffusion, and manufacturing with non-convexities. These superficially dissimilar market and non-market models share the common property that a decision maker's best "level of effort" depends positively upon other decision makers' "level of effort." This property is called strategic complementarity in the coordination failure literature.[2]

While it is tempting to assume that behavior will converge to an efficient equilibrium in situations with multiple Pareto ranked equilibria, doing so ignores the role of historical accident and dynamic process in producing mutually consistent behavior. Models of adaptive behavior often predict barriers that separate the space of outcomes into regions in which behavior does and regions in which behavior does not converge to an efficient equilibrium. The selected equilibrium is path dependent, that is, the equilibrium predicted to emerge depends on the historical accident of the initial condition, rather than on deductive concepts of efficiency.

---

[1] See Kreps (1990) for discussion, examples, and references.

[2] See Cooper and John (1988) and Milgrom and Roberts (1990) for examples and references.

The experiment reported in this paper was designed to discover if the predicted separation is observed. Our search was conducted in a generic game with strategic complementarities. While a deductive equilibrium analysis predicts multiple Pareto ranked equilibria, the observed behavior was systematic. We did observe the separation predicted by the fictitious play dynamic. Moreover, the equilibrium selected was sensitive to small differences in initial conditions as predicted. The experiment provides some striking examples of coordination failure growing from small historical accidents.

Finally, the paper introduces a measure of the origin of mutually consistent behavior. Our measure searches the space of ε-equilibria to find the value $\varepsilon(t)^*$ that maximizes Selten's (1991) measure of predictive success. Our measure reveals that, while initially naive subjects behave more like decision theorists than game theorists, their behavior becomes mutually consistent in the sense that $\varepsilon(t)^*$ is decreasing with time. While adaptive behavior leads to mutually consistent behavior, the mutually consistent behavior that emerges is path dependent.

II. ANALYTICAL FRAMEWORK.

To focus the analysis, consider the following generic version of a game with strategic complementarities and positive spillovers. Let $e^1, \ldots, e^n$ denote the actions taken by n players, where n is an odd number greater than one. Let e denote this action combination. An abstract market process is a mapping from the action space into a real number, the market outcome M.

The market outcome could represent market thickness, industry production, average market price, aggregate demand, or aggregate supply. In the coordination failure literature, the mean of the players' actions is a common example of an abstract market process, see Cooper and John (1988). As the number of players increases, the influence of an individual player on the mean goes to zero and in the limit an individual player can not influence the market outcome. In order to capture the anonymity of a many person economy without using enormous group sizes, we use the median, rather than the mean, for the abstract market process, see Van Huyck, Battalio, and Beil (1991) for additional motivation. Let $M(e)$ denote the median of e.

The game Γ is defined by the following payoff function and action space for each of n players indexed by i:

$$\pi\left(e^{i}, M(e)\right) = a\,M(e) - b\left[e^{i} - \sqrt[3]{M(e)}\right]^{2} + c, \qquad a > 0, b > 0, \quad (1)$$

where $e^i \in [-x, x]$ and $x > 1$. Assume that the players have complete information about the payoff functions and feasible actions and that this information is common knowledge.

In our experiment we used a finite version of game Γ. Finiteness simplifies our analysis in many important ways. Game *G*, which was used in the experiment, is given by payoff table *G*. It was derived from equation (1) using the following parameter values: $a = 26$, $b = 40$, $c = 86$. Let $\mathbf{E}^i = \{1,...,14\}$ index the elements of player *i*'s finite set of actions[3] and let $\mathbf{E} = \mathbf{E}^1 \times ... \times \mathbf{E}^n$ denote the set of all possible outcomes. There were seven players. Hence, **E** contained 105,413,504 elements.

{Insert payoff table *G* about here.}

A useful theory of strategic behavior would make accurate and precise predictions about human behavior in game *G*. The most precise theory, a *point theory*, would predict a single element of the set of all possible outcomes. The accuracy of a point theory interpreted literally is not likely to be high. Consequently, a point theory's predictions are often reinterpreted as the predicted central tendency of observed outcomes.[4] A less precise theory, an *area theory*, would predict a subset of all possible outcomes. An area theory that predicted the set of all possible outcomes, **E**, would be perfectly accurate, but not useful.

The extant theory of games may provide a useful theory of strategic behavior. A basic concept in game theory is individual rationality. In game *G*, *individual rationality* requires that player *i* choose a feasible action e^i* from his set of feasible actions $\mathbf{E}^i$ that maximizes

$$\sum_{e^{-i} \in E^{-i}} p(e^{-i})\pi\left(e^{i},\ M(e^{i}, e^{-i})\right),$$

where e^{-i} denotes $\{e^1,...,e^{i-1},e^{i+1},...,e^n\}$, $p(e^{-i}) \in \Delta(\mathbf{E}^{-i})$ is a subjective probability distribution over the actions of the other players, and $\Delta(\mathbf{E}^{-i})$ denotes the set of probability distributions on $\mathbf{E}^{-i}$. e^i* is a subjective best

---

[3] The action space was restricted to {-1.44, -1.22, -1.00, -0.78, -0.56, -0.33, -0.11, 0.11, 0.33, 0.56, 0.78, 1.00, 1.22, 1.44} and actions were relabeled from 1 to 14 respectively. Hence, $\mathbf{E}^i$ equals {1,...,14}.

[4] A *distribution* theory would characterize a probability distribution over the set of feasible outcomes, see for example Crawford (1995).

response to $p(e^{-i})$.

Knowledge that player i is individually rational rules out certain actions. Specifically, those actions that are not a subjective best response to any $p(e^{-i}) \in \Delta(\mathbf{E}^{-i})$. Such actions are strictly dominated actions. Let $\mathbf{E}^i_0 = \mathbf{E}^i$, and $\mathbf{E}^i_k = \{e^i \in \mathbf{E}^i_{k-1} \mid e^i$ is a subjective best response to some $p(e^{-i}) \in \Delta(\mathbf{E}_0^{-i})$ $\}$. By finiteness, there must exist a K such that $\mathbf{E}^i_k = \mathbf{E}^i_K$ for all $k \geq K$. The Cartesian product of the sets $\mathbf{E}^1_K, \ldots, \mathbf{E}^n_K$ are the action combinations consistent with the abstraction assumptions of common knowledge of G and of individual rationality and will be denoted the set of serially undominated action combinations, $U(\mathbf{E}) = \mathbf{E}^1_K \times \ldots \times \mathbf{E}^n_K$.[5]

The set of serially undominated action combinations for game G is large, since only actions 1 and 2 can be deleted, see payoff table G. For example, when n equals 7 there are 12^7 or 35,831,808 elements contained in $U(\mathbf{E})$. The abstraction assumptions of common knowledge of G and of individual rationality does not lead to a very precise theory.

Following Selten (1991), let the *precision* of a theory be equal to the number of predicted elements divided by the number of possible elements. This measure varies between 0 and 1. A score close to 0 indicates a precise theory. Predicting the set of all possible outcomes gives a measure of precision equal to 1. The measure of precision for $U(\mathbf{E})$ is 35,831,808 divided by 105,413,504 or 0.34.

Imposing a mutual consistency requirement reduces the number of outcomes predicted and, hence, increases the theory's precision. An action combination e^* constitutes an ε-*equilibrium*, if it satisfies the following mutual consistency condition:

$$\pi\big(e^{i}, M(e^{i}, e^{-i}*)\big) \leq \pi\big(e^{i}*, M(e*)\big) + \varepsilon \qquad (2)$$

for all $e^i \in \mathbf{E}^i$ and for all i. Let $N^{\epsilon}(\mathbf{E})$ denote the set of all action combinations that satisfy condition (2). A *mutual best response outcome* is an observed action combination that satisfies condition (2) with ε equal to zero.

The concept of an ε-equilibrium provides an area theory for game G. For example, there are 702 elements in $N^0(\mathbf{E})$, which is the set of mutual best response outcomes. Hence, $N^0(\mathbf{E})$ has a measure of precision equal to 0.000007.

However, most of the elements in $N^0(\mathbf{E})$ are asymmetric action combinations. These asymmetric action combinations involve coordinating

[5] See Brandenburger and Dekel (1989, p.51). In two person games, it is equivalent to rationalizability, see Bernheim (1984) and Pearce (1984).

on what are called "minimal contributing sets" in the public goods literature, where $\frac{n+1}{2}$ players determine the median and $\frac{n-1}{2}$ players best respond to that median. These asymmetric action combinations confront players with a difficult assignment problem. The players are symmetrically endowed in game *G*, but these action combinations require players to behave asymmetrically. The deductive equilibrium selection principle of symmetry rules out asymmetric equilibria.

Since *G* is a symmetric game, a symmetric action combination occurs when all players choose the same action. A symmetric action combination constitutes a *symmetric* ε-*equilibrium* if it satisfies condition (2). Game *G* has two symmetric strict equilibria: {3,...,3} and {12,...,12}. These equilibria are strictly Pareto ranked, that is, $\pi(3,3) < \pi(12,12)$. For brevity, we will call the payoff-dominated symmetric equilibria the 'low' equilibrium and the payoff-dominant symmetric equilibria the 'high' equilibrium. The high equilibrium, {12,...,12}, is the unique payoff-dominant symmetric Nash equilibrium and would be selected by any deductive equilibrium selection theory that gives precedence first to symmetry and then to efficiency, see Harsanyi and Selten (1988) for example. The mutual consistency condition combined with the selection principles of symmetry and payoff-dominance provides a point prediction: the high equilibrium.

Point theories based on deductive selection principles are often inaccurate. Van Huyck, Battalio, and Beil (1990, 1991) report experimental treatments in which subjects always coordinated on equilibrium points that were payoff-dominated by other strict symmetric equilibria.[6] Giving people common information about payoff functions, feasible strategies, and institutions does not, typically, induce common expectations about the outcome of the game.[7] Hence, it would not be surprising if observed behavior in game *G* failed to satisfy a mutual consistency condition.

III. Adaptive Behavior in the Repeated Game.

A vague appeal to adaptive behavior is often used to avoid explaining the origin of mutually consistent behavior. However, specific models of adaptive behavior do not always predict convergence to a symmetric efficient equilibrium. The historical accident that initiated the process may

---

[6] See also Beckman (1989), Cooper, et al. (1990), and Straub (1990).

[7] See also Smith (1990).

determine which, if any, equilibrium is selected. Let $G(T)$ denote the repeated game in which n players play G for T periods.

The most general model of adaptive behavior considered here is Milgrom and Roberts' (1991, p.86) concept of adaptive learning. A sequence of actions is consistent with *adaptive learning* if player i eventually chooses only actions that are nearly best responses to some probability distribution over his competitors' action combinations, where near zero probability is assigned to actions that have not been played for a sufficiently long time. If behavior is consistent with this concept of adaptive learning, then the observed sequence of actions will converge to the set of serially undominated action combinations, U(**E**).[8] As we have established, predicting that the sequence of observed action combinations will eventually be contained in U(**E**) is an imprecise prediction for game G.

While Milgrom and Robert's definition of adaptive learning is general enough to include Cournot's myopic best response dynamic and Brown's fictitious play dynamic, these very particular models of adaptive behavior make more precise predictions in game G. By *myopic best response* dynamic we mean that in periods $t > 1$ a player makes a best response to last period's median. By *fictitious play* dynamic we mean that in periods $t > 1$ a player makes a best response to the observed frequency of historical medians, that is, if the j^{th} median has occurred h_j times, then a subject playing according to fictitious play would select e^i to maximize $\sum_{j=1}^{14} \pi(e^i, j) h_j$.

Figure 1*a* graphs the myopic best response and figure 1*b* graphs the fictitious play dynamic starting from values of M_1 equal to $\{1,7,8,14\}$. As figures 1*a* and 1*b* illustrate, the fixed points 3 and 12 are locally stable under both dynamics, that is, for a sufficiently small perturbation the process returns to the fixed point. It is conventional to call the corresponding Nash equilibria $\{3,...,3\}$ and $\{12,...,12\}$ "stable." The figure is divided into the basin of attraction of the fixed point 3 consisting of $\{1,2,3,4,5,6,7\}$ and the basin of attraction of the fixed point 12 consisting of $\{8,9,10,11,12,13,14\}$. The line separating these two regions is called the *separatrix* below. The fictitious play dynamic predicts the same stable

[8] See also Bernheim (1984), Moulin (1986), and Gul (1990). Jordan (1991) and Kalai and Lehrer (1993) make stronger assumptions about the rationality and prior beliefs of the players and, consequently, can prove convergence to Nash equilibrium for a class of repeated games. Conversely, if the dynamic process is not based on individually rational behavior, then it is possible to get convergence to non-Nash outcomes, see Friedman and Rosenthal (1986) for example.

equilibria and gives the same basins of attraction as the myopic best response dynamic in game *G*.

{Insert figure 1a and 1b about here.}

The myopic best response and fictitious play dynamics both make precise predictions given an initial condition: If $M_1 \in \{1,2,3,4,5,6,7\}$, then M_t converges to 3. If $M_1 \in \{8,9,10,11,12,13,14\}$, then M_t converges to 12. The myopic best response dynamic converges within five periods and the fictitious play dynamic converges within twenty-two periods. The historical accident initiating the process selects an equilibrium. Moreover, around the separatrix small initial differences in M_1 can lead to large differences in the equilibrium selected. The experiment was designed to discover if the predicted separation is observed.

IV. Experimental Design.

The period game was described to the subjects using payoff table *G*, where the payoffs denote cents. The low equilibrium {3,...,3} pays each subject 60¢, while the high equilibrium {12,...,12} pays each subject 112¢. The experiment consists of 15 or 40 plays of the period game and this was announced in the instructions.

Eight subjects participated in each experiment. Seven subjects were active players whose actions determined the median. One subject was an inactive player who played against the reported median and would have replaced a bankrupt active player. The subjects were given a 500¢ interest free loan at the beginning of the session, which was repaid at the end of the session. Since the most a subject can lose in a period is 217¢, a subject can not go bankrupt in less than three periods. No subject ever went bankrupt, but if one had the inactive player would have replaced the bankrupt active player. The role of the inactive player was common information.

The instructions (available upon request) were read aloud to insure that the description of the game was common information. No pre-play negotiation was allowed. After each repetition of the period game, the median action was publicly announced and the subjects calculated their earnings for that period. The only common historical data available to the subjects was the median.

The subjects were undergraduate economics students attending Texas A&M University in the 1990 and 1992 spring semester. A total of 80 subjects participated in the ten sessions reported below. After reading the instructions, but before the session began, the students filled out a questionnaire to determine that they understood how to read the payoff table and how to calculate the median of seven numbers. If any subject made a mistake on the questionnaire, the instructions were read again.

The fifteen period sessions take less than one and a half hours to conduct. Consequently, subjects could earn significantly more than the minimum wage. For example, if subjects coordinate on an efficient equilibria--any combination of four subjects choosing 14 and three subjects choosing 13 or 12, then the average subject would earn $18.19.

V. EXPERIMENTAL RESULTS.

Subjects responded to payoff table *G* in a systematic way. Figure 2*a* reports the observed cumulative distribution of actions in period one for sessions one to ten. Of the 70 active subjects, no subject chose action 3, which corresponds to the low equilibrium, and only 5 subjects chose action 12, which corresponds to the high equilibrium. Hence, naive subjects do not seem to impose a mutual consistency condition on their behavior.

{Insert figures 2a and 2b about here.}

Only three subjects chose an action less than 6. Actions 6 and 7 were the modal actions and were chosen by 44 percent of the subjects. Action 6 insures a subject the highest payoff in the worst possible outcome, that is, it insures a payoff of at least 23¢, see payoff table *G*. Action 6 is denoted the *secure action* below.

In figure 2*a*, the thin vertical line at 5.5 divides the action space into actions less than the secure action and actions greater than or equal to the secure action. The Kolmogorov T statistic of 0.39 rejects the null hypothesis of a uniform cumulative distribution of initial actions at the one percent level, see Conover (1980, table A14). The distribution of actions in period one appears to reflect a concern for out-of-equilibrium payoffs by the subjects.

Action 7 is the best response to a diffuse prior over the median. It is easily computed by adding up the payoffs in each row. Occasionally, we find evidence of subjects making such calculations. Of course, the empirical distribution of the median is not diffuse, see figure 2*b*. The Kolmogorov T statistic of 0.50 rejects the null hypothesis of a uniform cumulative distribution of initial medians at the one percent level.

A similar phenomena occurs in Van Huyck, Battalio, and Beil (1991) and this earlier work lead us to expect the result reported here. Our rule of thumb is that naive subjects behave more like decision theorists than game theorists, that is, they are unlikely to impose mutual consistency conditions on their initial behavior. For whatever reason, the observed behavior results in medians distributed randomly about the separatrix.

Figure 2*b* reports the observed cumulative distribution of period 1 medians for sessions one to ten. Half of the initial medians fall below the separatrix and half fall above the separatrix. Denote sessions with an initial median in the basin of attraction of the fixed point 3, the *low sessions*, and sessions with an initial median in the basin of attraction of the fixed point 12, the *high sessions*.

Specifically, M_1 was a 7 in sessions one, two, six, seven and ten, an 8 in sessions eight and nine, a 9 in sessions four and five, and an 11 in session three, see table one. Hence, the five low sessions are one, two, six, seven and ten, and the five high sessions are three, four, five, eight and nine. Both the myopic best response and the fictitious play dynamic predict that the low sessions will converge to the low equilibrium, while the high sessions will converge to the high equilibrium.

{Insert table one about here.}

Inspecting figure three reveals that the median in the five high sessions does converge toward 12 and the median in the five low sessions does converge toward 3. Sessions three, four, and five even produce a median equal to or greater than 12 within five periods. The time series for the median in session four corresponds to the precise predictions made by the myopic best response dynamic in all fourteen periods. It is, however, the only session that does so. Moreover, none of the four sessions predicted to converge to the low equilibrium produces a median equal to 3 within five periods.

{Insert figure three about here.}

Occasionally, the median diverges from the predicted attractor. Of the 17 times the median diverges, 15 times it is to a higher (and higher paying) median. These violations of monotonic convergence seem to be caused by the subject's attempts to coordinate on more efficient but asymmetric equilibria that are close (in Euclidian distance) to the equilibrium selected by the models of adaptive behavior. For example, there are 6 instances when the median diverges from a 12 to a 13. Playing a 13 when the median is 12 costs 2 cents, but if playing 13 increases the median to 13 then a player gains 5 cents, see payoff table *G*. A similar flat spot exists above 3. However, playing 13 when the median is 3 costs 197 cents.

The resistance to the dynamic is most pronounced in the low sessions, see figure three. Naturally, subjects in a low session are more likely to resist the logic of the myopic best response and fictitious play dynamics than subjects in a high session, since the low sessions are converging to less efficient outcomes. Both models of adaptive behavior fail to capture the influence of efficiency on observed behavior.

Both the myopic best response dynamic and the fictitious play dynamic predict a separatrix between a median of 7 and 8 and both predict that behavior will move away from this barrier. The most compelling evidence for these naive models of adaptive behavior is the following: all ten sessions started with an initial median contained in the set {7,8,9,10,11}[9]; after period six none of the observed medians were contained in the set {7,8,9,10,11}; moreover, the median never crossed the separatrix, that is, subjects trapped in the low equilibrium's basin of attraction never escaped.

These sessions provide an example of coordination failure. If the subjects coordinate on an efficient but asymmetric equilibrium--any combination of four subjects choosing 14 and three subjects choosing 13 or 12, then the average subject would earn \$18.19 in a fifteen period session. None of the sessions converge to this asymmetric equilibrium, which is consistent with the symmetry assumption implicit in the myopic best response and fictitious play dynamics. In fact, none of the sessions ever produce a median of 14.

The average subject in the first fifteen periods of the five low sessions earned \$9.71, which is about 53 percent of their efficient earnings. The average subject in the first fifteen periods of the five high sessions earned \$15.57, which is about 86 percent of their efficient earnings. The small differences in the distribution of subjects' period one choices result in large differences in average earnings. Specifically, the average subject in a high session earns about \$6 (or 60 percent) more than the average subject in a low session.

A. Accuracy of Myopic Best Response and Fictitious Play Dynamics.

The myopic best response and fictitious play dynamics provide simple selection dynamics and we never observed the median crossing the predicted separatrix. Behavior really does get trapped in the low

---

[9] While a median of 7 can result from an asymmetric equilibria, the medians {8,9,10,11} are special in that they can not result from an equilibrium of $G(1)$. However, Benoit and Krishna (1985) and Friedman (1986) demonstrate that in finitely repeated games with multiple Pareto ranked equilibria repeated play increases the number of the equilibrium points. Asymmetric equilibria supported by trigger strategies can be constructed that yield $M_t \in \{8,9,10,11\}$ for $t < T$ in $G(T)$.

equilibrium's basin of attraction. Hence, there is a sense in which these models of adaptive behavior make remarkably accurate qualitative predictions. However, these models also make precise predictions about the evolution of play for both the median and for individual subjects. In this section we compare the accuracy of the one step ahead predictions made by the myopic best response dynamic and the fictitious play dynamic, that is, best responding to last period's median or to the historical frequency of medians.

In order to determine which model of adaptive behavior is more accurate we need a measure of accuracy. The *hit rate* of a theory is the relative frequency of correct predictions. Table two reports the hit rates of the two dynamics for both the median and individual actions for periods 2 to 15.

{Insert table two about here.}

Overall, the two dynamics predict the median about equally well with fictitious play correctly predicting 41 percent of the medians and the myopic best response dynamic correctly predicting 49 percent of the medians. Fictitious play does significantly better than the myopic best response dynamic for the low sessions, but their relative performance is reversed for the high sessions, because, as discussed previously, the low sessions converge slowly while the high sessions converge quickly. The prediction that the current median will be a best response to last period's median achieves a hit rate of 69 percent in the high sessions. Neither dynamic achieves a hit rate for the median above 50 percent in the low sessions.

If we focus on individual subject behavior rather than the median, the myopic best response dynamic out scores fictitious play for both the low and the high sessions. Specifically, fictitious play accurately predicts 311 of the 980 observations giving a hit rate of 32 percent, while the myopic best response dynamic accurately predicts 383 of the 980 observations giving a hit rate of 39 percent, see table three. However, neither dynamic organizes even fifty percent of the observations on individual behavior.

Our findings are consistent with Milgrom and Roberts' (1991, p.85) observation that intelligent people employ a variety of learning strategies and, hence, it may not be possible to give a detailed description of how people actually reach their decisions. Their more general model of adaptive learning guarantees convergence to U(**E**). A prediction that subjects will play an action from the set of serially undominated actions has a hit rate of 99.7 percent. However, the theory lacks precision in game *G*.

Moreover, it does not predict the influence of initial conditions on the subsequent separation of behavior that is such a striking feature of the data.

B. Measuring the Degree of Mutually Consistent Behavior.

In the fifteen period sessions, a mutual best response outcome was only observed in period 13 of sessions one and three and neither was a symmetric outcome. Sessions nine and ten were run for 40 periods in the hopes that subjects would start to implement mutual best response outcomes more frequently with additional experience. However, a mutual best response outcome was only observed in period 39 of session 9 and in period 40 of session 10.

Acknowledging the strategic uncertainty confronting players in game *G* might lead one to doubt whether subjects will coordinate on a mutual best response outcome initially, but as experience accumulates one might expect to observe behavior conforming to tighter mutual consistency conditions. Using the concept of a mutual ε-best response outcome, this intuition can be formalized as the prediction that ε goes to zero as experience accumulates.

Let $\varepsilon(t)^*$ be the ε in period *t* corresponding to $N^{\varepsilon}(\mathbf{E})$ that maximizes some measure of predictive success. Selten (1991) argues persuasively that a reasonable measure of predictive success is the difference between a theory's hit rate and a theory's precision. Recall that the hit rate of a theory is the relative frequency of correct predictions and that the precision of a theory is the relative size of the predicted region within the set of all possible outcomes. A successful theory will be accurate and precise, which means that Selten's difference measure of predictive success will be close to 1.

Let *S(t)* denote a sample of observed action combinations for period *t* and let # denote an operator that gives the number of elements in a set, then $\varepsilon(t)^*$ is the argument that maximizes

$$\frac{\#\{\ s \in S(t)\ \mid\ s \in N^{\varepsilon}(\mathrm{E})\ \}}{\#S(t)} - \frac{\#N^{\varepsilon}(\mathrm{E})}{\#\mathrm{E}}. \tag{3}$$

A mutual $\varepsilon(t)^*$-best response outcome trades off some of the accuracy of the set of serially undominated action combinations, U(**E**), for some of the precision of the set of mutual best response action combinations, $N^0(\mathbf{E})$.[10]

The measure of predictive success for $U(\mathbf{E})$ and $N^0(\mathbf{E})$ provide useful benchmarks. In period 15, 9 of the 10 outcomes are contained in $U(\mathbf{E})$ and so the hit rate is 0.9. As mentioned above, the precision is 0.34. Hence, the measure of predictive success for $U(\mathbf{E})$ is 0.56. Since none of the outcomes in period 15 were mutual best response outcomes, the measure of predictive success for $N^0(\mathbf{E})$ is negative.

Table three reports the hit rate, precision, and predictive success of $N^\varepsilon(\mathbf{E})$ for values of ε from 0 to 15 cents. A value of ε equal to 8 cents results in a measure of predictive success equal to 0.799, which is larger than the measure of predictive success for $U(\mathbf{E})$ even though it has a lower hit rate. A value of ε equal to 14 cents maximizes the measure of predictive success at 0.996. Hence, ε(15)* equals 14 cents. Imposing a mutual consistency condition leads to a more precise theory that is also more accurate than one based on dominance arguments alone.

{Insert table three about here.}

Figure four reports the time series of ε(*t*)* for the five high and five low sessions. Both the high and low sessions begin with about the same value of ε(1)* as would be expected given the absence of any shared experience in game *G*. Specifically, ε(1;high)* equals 92 cents and ε(1;low)* equals 88 cents. The ε(*t*;high)* then drops significantly below ε(*t*;low)* for the next four periods. This seems to reflect a greater willingness by subjects to follow the logic of adaptive behavior when it is moving to a more efficient equilibrium than to a less efficient equilibrium. After period 6, however, there does not seem to be any particular relationship between our measure of mutually consistent behavior and the initial condition.

{Insert figure four about here.}

The relatively high values of ε(*t*)* at the end of the fifteen period sessions led us to wonder if increasing the number of periods would result in ε(*t*)* continuing to decrease toward 0. Hence, we ran sessions nine and ten for forty periods. Figure five reports the time series of ε(*t*)* for session nine and session ten independently. Recall that session nine is a high

---

[10] Since (3) restricts the elements in the set of predicted action combinations to be ε-equilibria of the period game, it does not give an unimprovable area theory, see Selten (1991) or McCabe, Rassenti, and Smith (1991).

session and session ten is a low session. Figure five has the same essential pattern for the first fifteen periods as appeared in the pooled data reported in figure four.

{Insert figure five about here.}

The value of $\varepsilon(t)^*$ does appear to be slowly decreasing. For example, after period 25 the value of $\varepsilon(t)^*$ is always below 10 cents for both series. Moreover, after period 28 the value of $\varepsilon(t;9)^*$ is always equal to or less than 5 cents and after period 32 it is always equal to or less than 2 cents.

Whether $\varepsilon(t)^*$ would have converged to zero without the announced terminal period is an open question. The subjects were not particularly successful at coordinating on asymmetric mutual best response outcomes; but, clearly, the existence of more efficient, but asymmetric, equilibria close (in Euclidian distance) to the stable fixed points of the dynamical systems retards convergence to a mutual best response outcome. Our results lead us to believe that adaptive behavior will not only converge to the set of serially undominated actions, but will also converge to an outcome that satisfies a mutual consistency condition like that imposed in an ε-equilibrium if not a Nash equilibrium.

VI. Summary and Conclusion.

This paper has reported on subjects' adaptive behavior in a generic game with strategic complementarities. Both the myopic best response dynamic and the fictitious play dynamic predict a separatrix between a median of 7 and 8 and both predict that behavior will move away from this barrier. Given the high precision of these models of adaptive behavior, they make remarkably accurate predictions. Specifically, all ten sessions started with an initial median contained in the set {7,8,9,10,11}; after period six none of the observed medians were contained in the set {7,8,9,10,11}; moreover, the median never crossed the separatrix, that is, subjects trapped in the low equilibrium's basin of attraction never escaped.

Initially, subjects behave more like decision theorists than game theorists in the sense that a concern for security rather than mutual consistency appears to influence their choices. For example, 44 percent of the subjects chose their secure action or their best response to a diffuse prior over the median in the first period of the ten sessions. We expected this and designed payoff table *G* to exploit subjects' systematic response to security in order to generate initial conditions that are close to the separatrix without having to resort to crude methods, like forced trials, etc.

Over time subjects' behavior becomes more consistent. Using the concept of a mutual ε-best response outcome to measure the degree of mutually consistent behavior, we found that ε equal to 14 cents after 15 periods and to 2 cents after 40 periods maximized Selten's measure of predictive success. This convergence to mutually consistent behavior did not necessarily lead to more efficient outcomes.

The average subject in the first fifteen periods of the five low sessions earned $9.71. The average subject in the first fifteen periods of the five high sessions earned $15.57. The small differences in the distribution of subjects' period one choices result in large differences in average earnings. Specifically, the average subject in a high session earns about $6 (or 60 percent) more than the average subject in a low session.

The myopic best response dynamic had a slightly higher hit rate than the fictitious play dynamic. However, game $G(T)$ is not well suited for discriminating between the myopic best response dynamic and the fictitious play dynamic, since both predict the same attractors and the same basins of attraction: see Van Huyck, Cook, and Battalio (1994) for an environment in which fictitious play easily does better than myopic best response dynamics. Whether it is possible to create an accurate and precise model of adaptive behavior in repeated games remains an open question. Our results suggest that at least for some strategic situations it should be possible to construct accurate models of adaptive behavior that are much more precise than conventional wisdom suggests.

The following analogy may help explain our discovery. Mark Twain (1962, p.86) describes a remarkable spring at the summit of a Rocky Mountain pass that "spent its water through two outlets and set it in opposite directions." One of the streams starts a journey westward to the Gulf of California and the Pacific Ocean. The other starts a journey eastward to the Gulf of Mexico and the Atlantic Ocean. Our search was for a spring that straddles a barrier dividing a continent of human behavior. Payoff table *G* is such a spring and either the myopic best response dynamic or the fictitious play dynamic predict such a barrier. Unlike the Rocky Mountains, this barrier is hard to see without a vision informed by models of adaptive behavior.

References

Beckman, Steve, "Producer's Dilemma Experiments," *Journal of Economic Behavior and Organization*, 11 (1989), 27-46.

Benoit, J. and V. Krishna, "Finitely Repeated Games," *Econometrica*, 53(4), July 1985, 905-922.

Bernheim, B. Douglas, "Rationalizable Strategic Behavior," *Econometrica*, 52(4), July 1984, 1007-1028.

Brandenburger, A. and E. Dekel, "The Role of Common Knowledge Assumptions in Game Theory," in *The Economics of Missing Markets, Information, and Games*, edited by F. Hahn, (Oxford: Clarendon Press, 1989).

Brown, G.W., "Iterative Solution of Games by Fictitious Play," in *Activity Analysis of Production and Allocation*, T. Koopmans, ed., (New York: Wiley, 1951), 374-376.

Conover, W.J., *Practical Nonparametric Statistics, second edition*, (New York: John Wiley & Sons, 1980).

Cooper, Russell, Douglas V. DeJong, Robert Forsythe, and Thomas W. Ross, "Selection Criteria in Coordination Games: Some Experimental Results," *The American Economic Review*, 80(1), March 1990.

Cooper, Russell and Andrew John, "Coordinating Coordination Failures in Keynesian Models," *The Quarterly Journal of Economics*, 103(3), August 1988, 441-464.

Cournot, Augustine, *Researches into the Mathematical Principles of the Theory of Wealth*, (London, Hafner, 1960).

Crawford, Vincent P., "Adaptive Dynamics in Coordination Games," *Econometrica*, 63(1), January 1995, 103-145.

Gul, Faruk, "Rational Strategic Behavior and The Notion of Equilibrium," working paper, January 1990.

Harsanyi, John and Reinhard Selten, *A General Theory of Equilibrium Selection in Games*, (Cambridge, MIT Press, 1988).

Friedman, James W., *Game Theory with Applications to Economics*, (New York: Oxford University Press, 1986.)

Friedman, James W., and Robert W. Rosenthal, "A Positive Approach to Non-Cooperative Games," *Journal of Economic Behavior and Organization*, 7, 1986, 235-251.

Jordan, James, "Bayesian Learning in Normal Form Games," *Games and Economic Behavior*, 3(1), February 1991, 60-81.

Kalai, Ehud and Ehud Lehrer, "Rational Learning Leads to Nash Equilibrium," *Econometrica*, 61(5), September 1993, 1019-1045.

Kreps, David M., *Game Theory and Economic Modelling*, (Oxford,UK: Clarendon Press, 1990).

Lucas, Robert E., Jr., "Adaptive Behavior and Economic Behavior," in *Rational Choice: the contrast between economics and psychology*, edited by R. Hogarth and M. Reder, (Chicago, University of Chicago Press, 1987).

McCabe, Kevin, Stephen Rassenti, and Vernon Smith, "Cooperation and the Repeated Interactions of Anonymous Pairings," working paper, October 1991.

Milgrom, Paul and John Roberts, "Adaptive and Sophisticated Learning in Normal Form Games," *Games and Economic Behavior*, 3(1), February 1991, 82-100.

Moulin, Herve, *Game Theory for the Social Sciences*, (New York, NY: New York University Press, 1986).

Pearce, David G., "Rationalizable Strategic Behavior and the Problem of Perfection," *Econometrica*, 52(4), July 1984, 1029-1050.

Selten, Reinhard, "Properties of a Measure of Predictive Success," *Mathematical Social Sciences*, 21, 1991, 153-167.

Smith, Vernon L., "Experimental Economics: Behavioral Lessons for Microeconomic Theory and Policy," 1990 Nancy L. Schwartz Memorial Lecture, Northwestern University.

Straub, Paul G., "Understanding Coordination Failures," working paper, October 1990.

Twain, Mark, *Roughing It*, (New York, New American Library, Inc., 1962).

Van Huyck, John B., Raymond C. Battalio, and Richard O. Beil, "Tacit Coordination Games, Strategic Uncertainty, and Coordination Failure," *The American Economic Review* 80(1), March 1990, 234-248.

Van Huyck, John B., Raymond C. Battalio, and Richard O. Beil, "Strategic Uncertainty, Equilibrium Selection, and Coordination Failure in Average Opinion Games," *The Quarterly Journal of Economics*, 106(3), August 1991, 885-910.

Van Huyck, John B., Joseph P. Cook, and Raymond C. Battalio, "Selection Dynamics, Asymptotic Stability, and Adaptive Behavior," *Journal of Political Economy,* 102(5), October 1994, 975-1005.

TABLE ONE

Subject Choices Periods 1 to 15 by Session:
Payoff Table G.

Subject	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Session 1															
1:	6	7	8	6	5	7	4	4	4	4	4	5	4	4	3
2:	5	7	9	7	4	4	3	3	3	5	5	3	3	3	3
3:	10	6	8	7	5	4	4	5	4	3	3	3	3	3	4
4:	9	5	4	5	5	5	4	4	4	4	4	4	4	3	4
5:	10	6	6	5	5	5	4	5	4	4	5	4	4	4	4
6:	6	7	5	6	5	6	5	7	5	6	5	5	4	3	5
7:	7	6	5	5	5	5	4	3	3	3	3	3	3	3	3
Median:	7	6	6	6	5	5	4	4	4	4	4	4	4*	3	4
Session 2															
1:	12	12	6	5	5	4	4	4	4	4	4	4	3	3	3
2:	8	8	5	4	4	3	3	3	3	3	3	3	3	3	3
3:	6	6	6	5	6	6	5	5	5	4	4	4	4	4	4
4:	7	6	5	8	6	6	7	4	5	4	4	3	4	3	3
5:	6	6	6	5	5	4	5	5	5	5	5	5	5	5	5
6:	11	10	6	5	4	4	3	5	3	3	3	3	3	3	3
7:	7	7	7	4	4	4	4	3	3	3	3	3	3	3	3
Median:	7	7	6	5	5	4	4	4	4	4	4	3	3	3	3
Session 3															
1:	13	12	14	14	12	13	14	12	12	13	13	13	13	13	12
2:	7	8	9	11	12	11	11	10	12	12	12	12	12	12	12
3:	14	14	14	13	14	13	14	14	14	14	14	14	13	13	13
4:	6	9	10	14	12	12	12	12	12	12	12	12	12	12	12
5:	14	14	14	13	13	13	13	13	13	13	13	13	13	13	13
6:	11	12	13	13	12	14	13	12	13	14	14	13	13	12	12
7:	6	9	11	11	14	10	11	13	12	12	11	14	12	14	12
Median:	11	12	13	13	12	13	13	12	12	13	13	13	13*	13	12
Session 4															
1:	4	10	10	10	13	12	12	12	12	12	12	12	12	12	12
2:	12	10	11	11	12	12	13	12	12	12	12	12	12	12	12
3:	7	12	4	14	14	10	11	12	5	12	10	10	10	7	11
4:	6	11	11	13	12	12	13	12	12	13	13	13	13	13	13
5:	10	6	8	12	9	13	12	11	12	12	12	12	12	13	12
6:	11	11	12	11	12	12	12	12	12	12	12	12	12	12	12
7:	9	8	11	12	13	12	12	12	12	12	14	14	14	14	12
Median :	9	10	11	12	12	12	12	12	12	12	12	12	12	12	12
Session 5															
1:	8	8	8	8	11	12	12	12	12	13	12	12	12	12	12
2:	7	8	11	12	13	11	12	13	13	13	11	11	12	13	12
3:	11	12	11	12	12	12	12	12	12	12	12	12	12	12	12
4:	13	12	14	14	14	14	13	13	13	13	13	13	13	13	13
5:	6	7	6	10	10	10	10	9	12	12	12	12	12	12	12
6:	9	6	6	9	12	12	14	12	12	12	12	12	12	12	12
7:	10	11	12	12	11	12	11	12	12	12	12	12	12	12	12
Median:	9	8	11	12	12	12	12	12	12	12	12	12	12	12	12

TABLE ONE (Continued)

Subject	Period 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Session 6															
1:	9	8	7	5	6	5	5	5	6	5	5	6	5	5	6
2:	6	6	6	7	6	7	7	6	6	6	6	6	6	6	6
3:	7	6	9	7	10	7	5	6	7	6	7	6	6	6	7
4:	10	14	8	9	3	7	6	4	3	7	5	5	7	4	5
5:	6	6	6	7	7	6	6	7	6	6	5	6	5	5	5
6:	7	8	6	6	8	6	6	5	4	4	5	4	6	4	6
7:	8	4	4	4	5	5	5	5	4	4	4	3	4	4	4
Median:	7	6	6	7	6	6	6	5	6	6	5	6	6	5	6
Session 7															
1:	6	7	5	5	7	7	6	3	3	4	5	5	6	5	3
2:	6	5	6	6	6	5	5	4	5	5	6	5	6	6	6
3:	10	9	5	5	4	5	7	5	6	6	5	4	4	4	4
4:	7	7	7	8	7	6	6	6	4	5	6	5	5	5	5
5:	7	7	7	8	5	4	4	5	8	8	8	6	4	4	4
6:	6	6	5	6	6	5	5	4	6	6	5	6	6	5	5
7:	10	3	6	9	2	5	4	1	7	8	3	6	5	2	1
Median:	7	7	6	6	6	5	5	4	6	6	5	5	5	5	4
Session 8															
1:	7	8	6	10	11	11	12	12	12	12	12	12	12	12	12
2:	12	8	9	10	11	11	11	11	12	12	12	12	12	12	12
3:	7	9	9	12	11	13	13	13	13	13	13	13	13	13	13
4:	12	12	13	13	13	13	13	14	14	13	13	14	14	14	14
5:	12	9	12	10	12	12	13	13	13	13	13	13	13	13	13
6:	7	9	11	13	12	12	10	13	13	12	12	13	12	13	13
7:	8	11	11	11	11	12	12	12	12	12	12	12	12	12	12
Median:	8	9	11	11	11	12	12	13	13	12	12	13	12	13	13
Session 9															
1:	14	9	9	9	10	11	12	12	12	12	12	12	12	13	12
2:	8	7	7	8	8	8	9	9	9	9	9	9	9	10	10
3:	8	7	6	9	7	9	10	10	11	11	12	12	12	12	12
4:	6	9	8	10	10	11	12	12	12	12	12	12	12	13	13
5:	8	10	11	10	10	11	14	13	12	12	12	12	12	12	12
6:	14	14	13	13	13	13	13	12	12	12	12	12	12	12	12
7:	7	7	7	8	8	9	8	9	9	11	12	13	13	12	12
Median:	8	9	8	9	10	11	12	12	12	12	12	12	12	12	12
Session 10															
1:	9	4	5	5	4	3	3	3	3	3	3	5	4	4	4
2:	5	5	4	4	3	3	3	4	5	4	5	5	6	6	6
3:	11	7	10	7	11	6	5	5	5	4	4	4	4	4	4
4:	10	12	5	4	5	5	7	3	3	5	5	6	7	5	4
5:	7	5	3	4	3	4	4	1	3	3	3	3	5	5	4
6:	7	6	4	6	5	5	3	4	3	3	4	5	4	5	4
7:	6	5	4	4	4	3	3	5	3	4	5	4	5	6	5
Median:	7	5	4	4	4	4	3	4	3	4	4	5	5	5	4

* denotes a mutual best response outcome.

TABLE TWO
Myopic Best Response Dynamic versus Fictitious Play Dynamic
(Periods 2 to 15)

	Low Sessions (1, 2, 6, 7, 10)		High Sessions (3, 4, 5, 8, 9)		Total	
	Hits	Percent	Hits	Percent	Hits	Percent
Hit Rate for Median						
Fictitious Play	29	(41)	29	(41)	58	(41)
Myopic Best Response	21	(30)	48	(69)	69	(49)
Total Possible	70	(100)	70	(100)	140	(100)
Hit Rate for Actions						
Fictitious Play	138	(28)	173	(35)	311	(32)
Myopic Best Response	191	(39)	192	(39)	383	(39)
Total Possible	490	(100)	490	(100)	980	(100)

TABLE THREE
Predictive Success of $N^{\varepsilon}(\mathbf{E})$
Period 15 of Sessions 1 to 10

ε	Hit Rate	Precision	Predictive Success
0	0.0	0.000007	-0.000007
1	0.0	0.000012	-0.000012
2	0.3	0.000031	0.299969
3	0.3	0.000047	0.299953
4	0.3	0.000060	0.299940
5	0.5	0.000175	0.499825
6	0.5	0.000239	0.499761
7	0.5	0.000395	0.499605
8	0.8	0.000848	0.799152
9	0.8	0.001047	0.798953
10	0.8	0.001085	0.798915
11	0.8	0.001809	0.798191
12	0.8	0.001863	0.798137
13	0.8	0.002604	0.797396
14	1.0	0.003409	0.996591
15	1.0	0.003789	0.996211

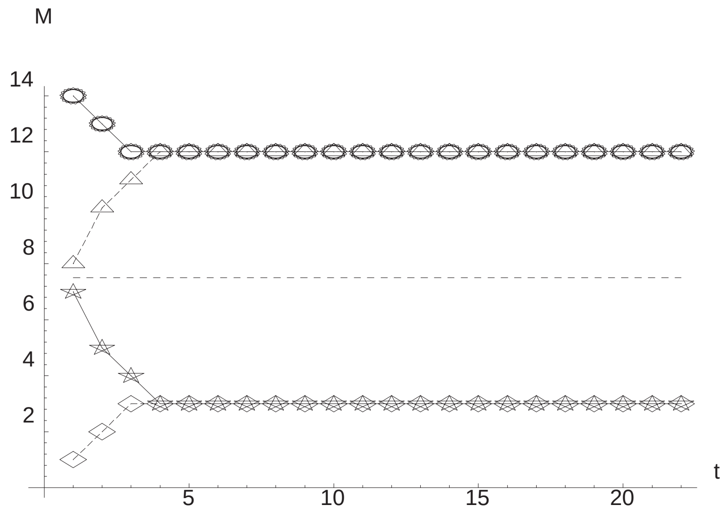

Figure 1a: Myopic best response dynamic and separatrix

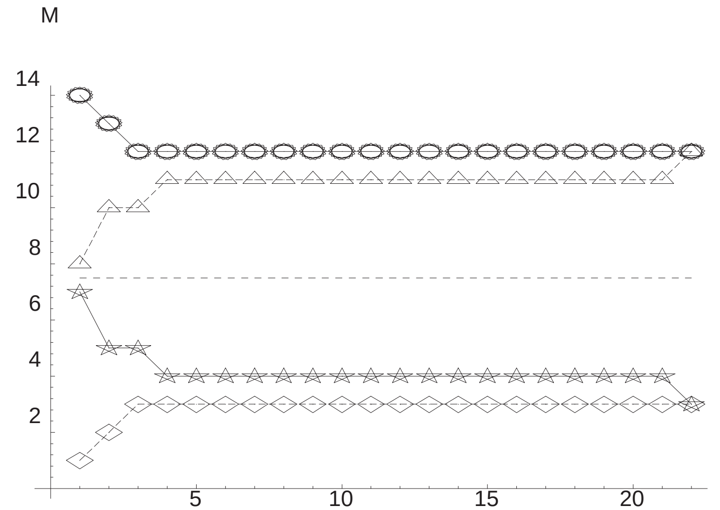

Figure 1b: Fictitious play dynamic and separatrix. (Paths chosen for slowest convergence when best response was not unique.)

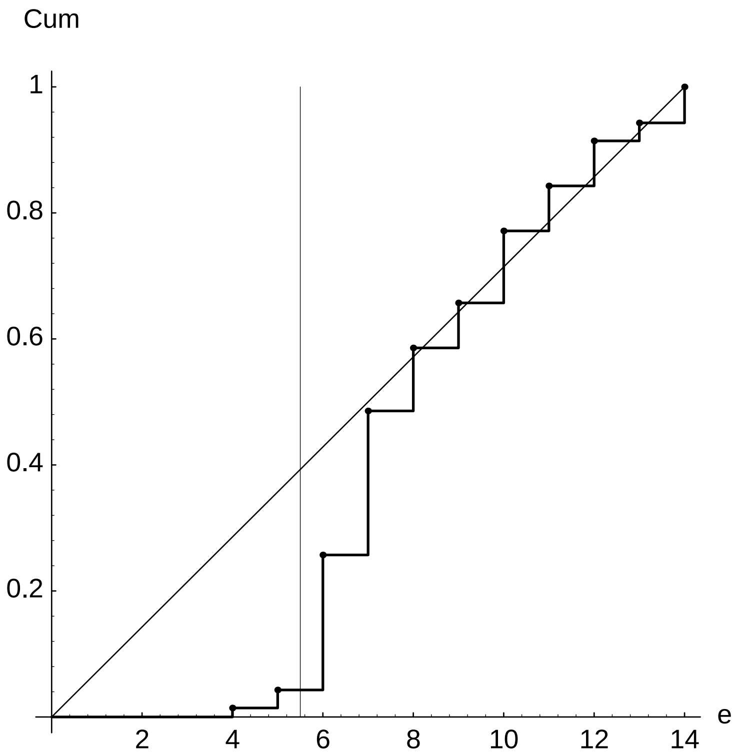

Figure 2a: Observed cumulative distribution of period 1 actions for sessions 1 to 10. The Kolmogorov T statistic of 0.39 rejects the null hypothesis of a uniform cumulative distribution at the one percent level.

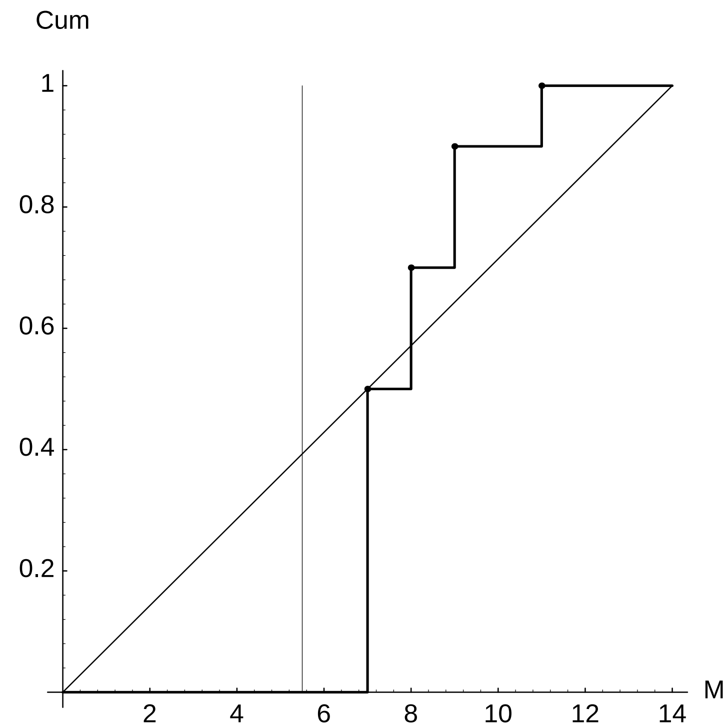

Figure 2b: Observed cumulative distribution of period 1 medians for sessions 1 to 10. The Kolmogorov T statistic of 0.50 rejects the null hypothesis of a uniform cumulative distribution at the one percent level.

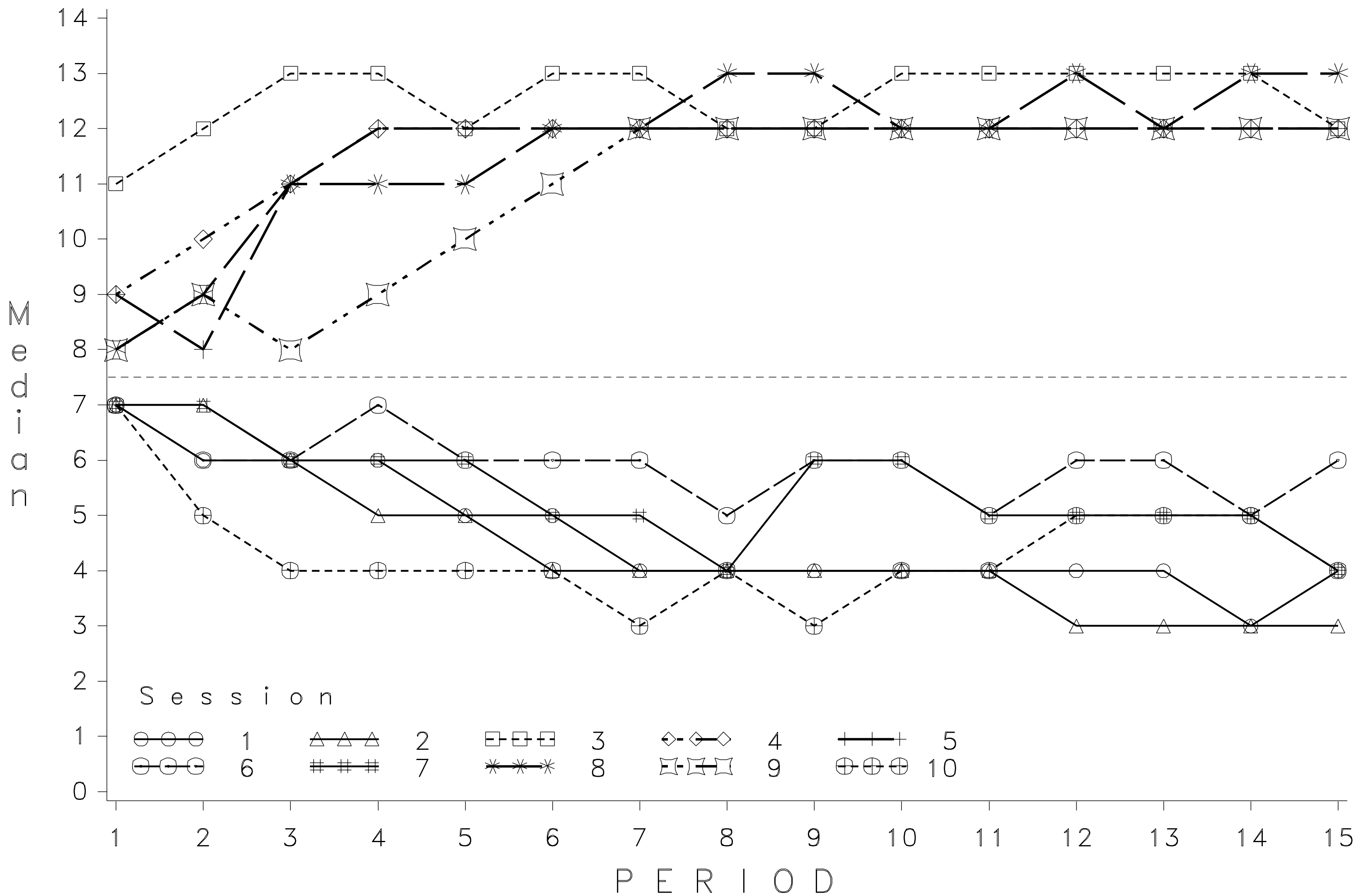

Figure 3: Median choice in sessions 1 to 10 by period.

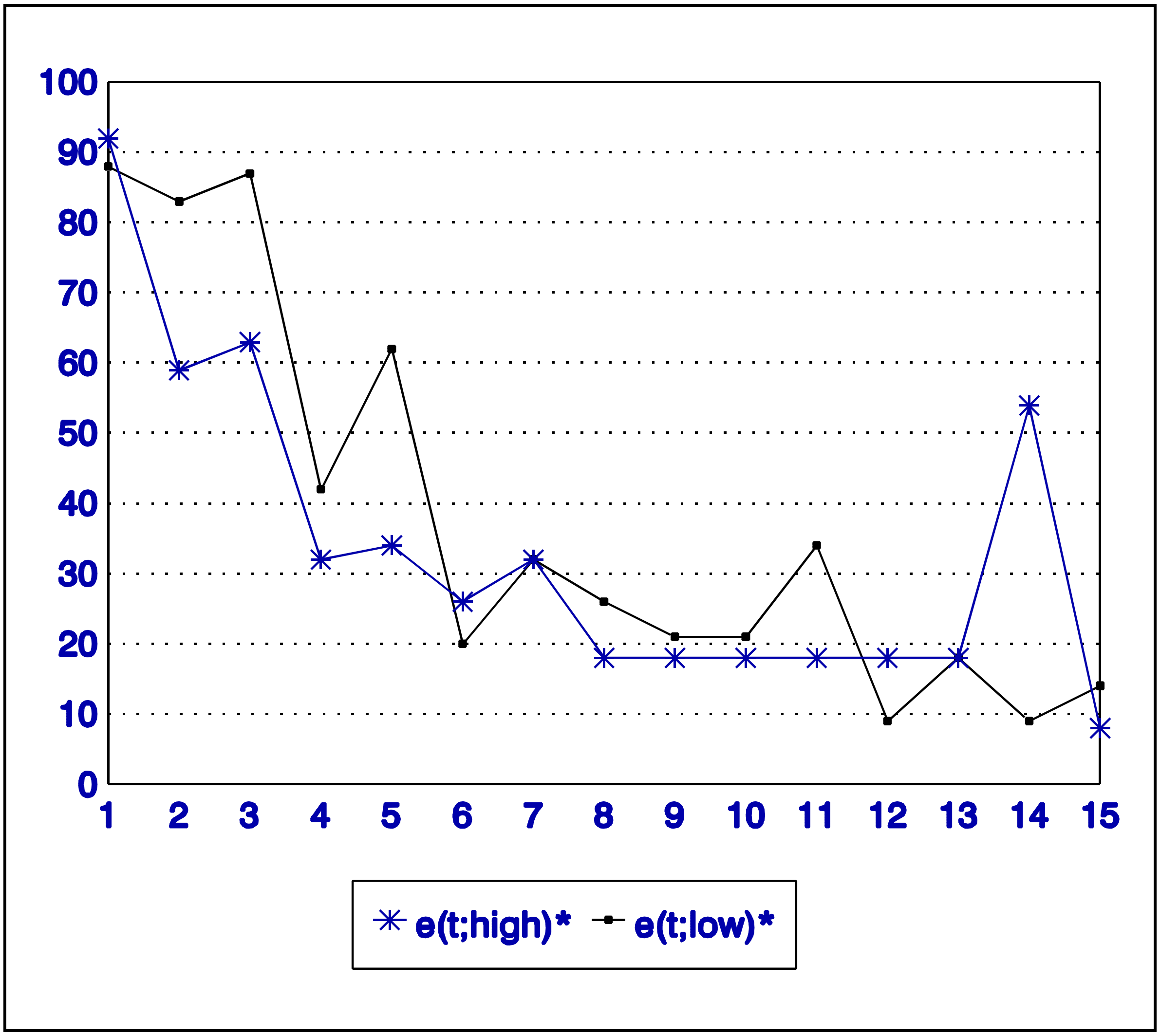

Figure 4: Time Series of e(t)* for high and low sessions, Periods 1 to 15. (High denotes sessions 3,4,5,8,9. Low denotes sessions 1,2,6,7,10.

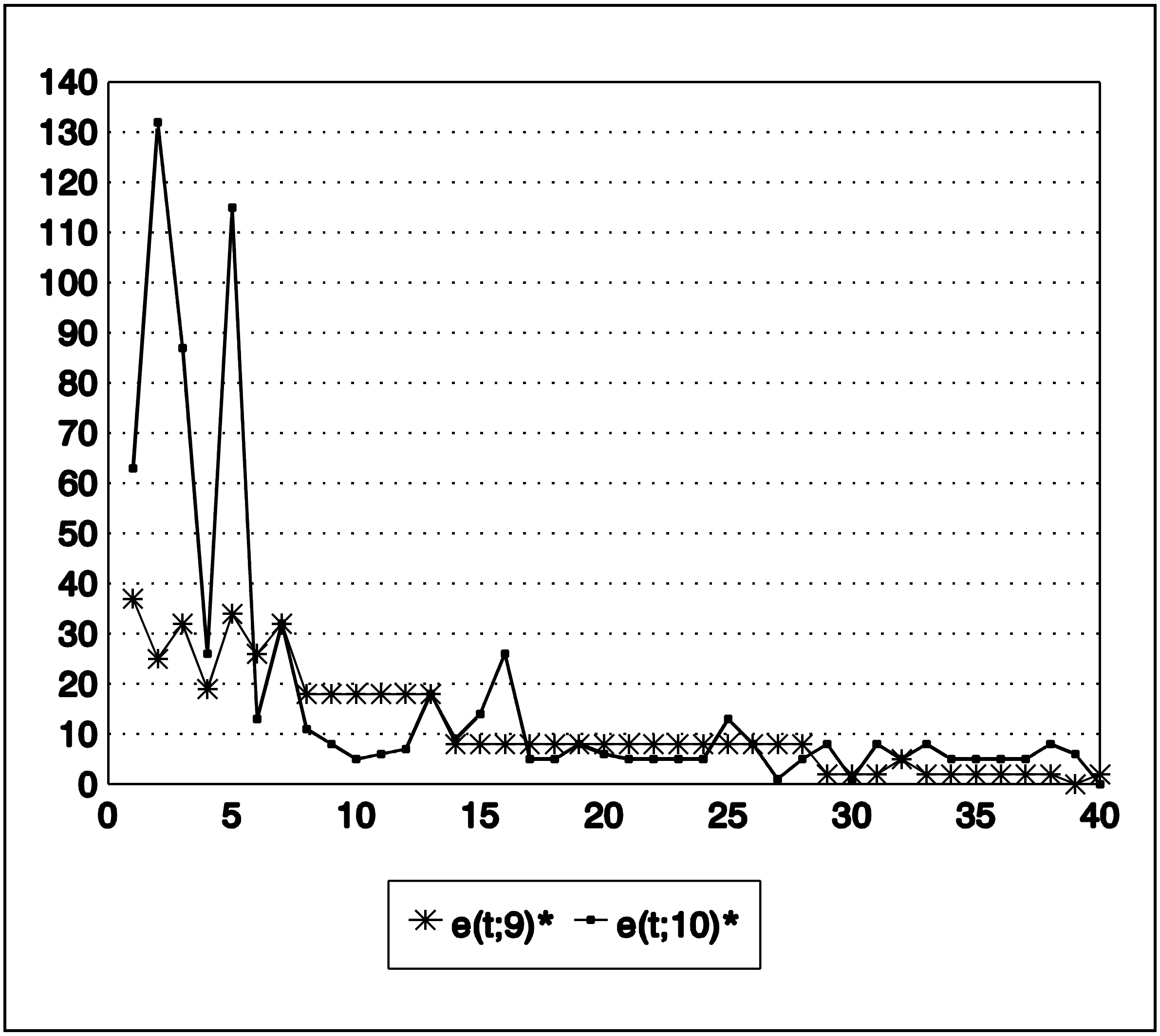

Figure 5: Time Series for e(t)* sessions 9 and 10, Periods 1 to 40.

Payoff Table G

Your Choice / Median Choice

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	45	49	52	55	56	55	46	-59	-88	-105	-117	-127	-135	-142
2	48	53	58	62	65	66	61	-27	-52	-67	-77	-86	-92	-98
3	48	54	60	66	70	74	72	1	-20	-32	-41	-48	-53	-58
4	43	51	58	65	71	77	80	26	8	-2	-9	-14	-19	-22
5	35	44	52	60	69	77	83	46	32	25	19	15	12	10
6	23	33	42	52	62	72	82	62	53	47	43	41	39	38
7	7	18	28	40	51	64	78	75	69	66	64	63	62	62
8	-13	-1	11	23	37	51	69	83	81	80	80	80	81	82
9	-37	-24	-11	3	18	35	57	88	89	91	92	94	96	98
10	-65	-51	-37	-21	-4	15	40	89	94	98	101	104	107	110
11	-97	-82	-66	-49	-31	-9	20	85	94	100	105	110	114	119
12	-133	-117	-100	-82	-61	-37	-5	78	91	99	106	112	118	123
13	-173	-156	-137	-118	-96	-69	-33	67	83	94	103	110	117	123
14	-217	-198	-179	-158	-134	-105	-65	52	72	85	95	104	112	120